AGH University of Krakow

Faculty of Physics and Applied Computer Science

MODULE 7

# LIMITATIONS AND OPEN QUESTIONS OF THE STANDARD MODEL

Supporting Lecture Notes for *The Standard Model*

**The Guideline**

*The aim of this expanded module note is to make visible why the Standard Model must be studied not only as a spectacularly successful theory, but also as a theory with clear internal boundaries. Starting from the logic of discrete symmetries, flavour mixing, and CP violation, the note develops the quark-sector origin of Standard Model CP violation, its phenomenological manifestations, and the reason why it does not suffice to explain the observed baryon asymmetry of the Universe. The note then turns to other major open issues - neutrino masses, the flavour and Yukawa hierarchy puzzle, dark matter, the strong CP problem, and naturalness questions associated with the electroweak scale - always with the same pedagogical aim: to distinguish what the Standard Model explains, what it merely parametrises, and what it fails to account for. Throughout, the emphasis is on treating Module 7 not as a full course on physics beyond the Standard Model, but as the carefully bounded bridge between mastery of the Standard Model and a clear understanding of why present-day particle physics must still look beyond it.*

Prepared for the Standard Model course

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# Preface

These supporting notes are written for *Module 7: Limitations and Open Questions of the Standard Model* of the AGH course *The Standard Model*. The guiding question of the module is not whether the Standard Model works — it plainly does, across an enormous range of phenomena — but how one should understand the fact that a successful theory can still be incomplete. By the time one reaches this module, the Standard Model has already appeared as a tightly constrained relativistic quantum field theory with gauge structure, spontaneous symmetry breaking, and a predictive phenomenology of decays, cross sections, and observables. The natural next step is therefore reflective rather than destructive: one must ask which features of nature are genuinely explained by the theory, which are merely encoded in its free parameters, and which established facts already lie beyond the minimal Standard Model description.

The pedagogical logic of the module is

$$\text{Standard Model success} \longrightarrow \text{discrete symmetries and CP} \longrightarrow \text{flavour structure and CKM mixing} \longrightarrow \text{observed CP viola}$$

This chain is deliberate. The module does not begin with speculative model building. It begins inside the Standard Model itself, with one of the places where the theory is both most successful and most revealing: flavour physics and CP violation. Only after this internal discussion does the note move outward to questions such as neutrino masses, dark matter, and naturalness, which show in different ways that the Standard Model cannot be the final description of fundamental physics.

The distinction between *limitations of the Standard Model* and *physics beyond the Standard Model* is central. A limitation may mean different things. It may mean empirical incompleteness, as in the case of dark matter. It may mean direct failure of the minimal renormalisable Standard Model field content, as in the case of neutrino masses. It may mean an explanatory shortfall, as in the baryon asymmetry problem. Or it may mean a conceptual tension, as in naturalness and hierarchy questions. Physics beyond the Standard Model begins when one starts proposing new dynamical principles, new fields, or new ultraviolet completions to address such issues. The purpose of Module 7 is not to survey those proposals in detail, but to sharpen the questions that motivate them.

This note is intentionally placed after the Higgs-sector and phenomenology modules. Module 5 showed how the Standard Model can accommodate masses without abandoning gauge symmetry. Module 6 showed how the theory turns into amplitudes, rates, and observables. Module 7 now asks what remains unexplained even after these achievements are granted. In that sense the present module is both retrospective and forward-looking. It closes the conceptual arc of the Standard Model course while simultaneously preparing the student to understand why current particle-physics research continues to search for deeper structure beyond the Standard Model itself.

# Conventions and notation

- Metric signature: $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$.
- Natural units: $\hbar = c = 1$.
- Discrete symmetries are denoted by $C$, $P$, $T$, $CP$, and $CPT$ in the standard way.
- Quark weak-eigenstate and mass-eigenstate misalignment is encoded in the CKM matrix $V_{\text{CKM}}$.
- When the lepton sector is mentioned, neutrino mixing is denoted by the PMNS matrix $U_{\text{PMNS}}$.

- Baryon and lepton number are written as $B$ and $L$.
- The electroweak Higgs vacuum expectation value is denoted by $v$.
- Yukawa couplings are denoted generically by $y_f$; after electroweak symmetry breaking, charged-fermion masses satisfy $m_f \sim y_f v$.
- The QCD vacuum angle relevant to the strong CP problem is denoted by $\bar{\theta}$.
- The dual field-strength tensor is written as

$$\widetilde{G}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}.$$

- When dark matter is mentioned schematically, its cosmological abundance is indicated by $\Omega_{\mathrm{DM}}$.
- Matter–antimatter asymmetry is discussed qualitatively; when needed, the baryon asymmetry is denoted schematically by a nonzero baryon excess or by a baryon-to-photon asymmetry parameter.

# 1 Introduction and module roadmap

## 1.1 Why a limitations module belongs in a Standard Model course

A course on the Standard Model should not end with the false impression that scientific maturity consists only in listing successes. The Standard Model is indeed one of the most powerful theoretical structures in modern physics. It explains the pattern of gauge interactions, the role of chirality in the weak interaction, the mechanism of electroweak symmetry breaking, and a vast array of decay and scattering data. Yet precisely because it works so well, students must also learn to ask a sharper question: what kind of theory is it, and in what sense is it complete or incomplete?

This is not an external or optional question. It belongs to the internal pedagogy of the subject. A theory can be predictive and yet contain unexplained structures. It can describe observed data accurately and yet fail to account for some established phenomena. It can even remain mathematically consistent over the energies presently tested and still look implausibly provisional when viewed as a candidate for a deeper theory of nature. Module 7 therefore belongs inside a Standard Model course because understanding a theory includes understanding the boundary between what it genuinely explains and what it leaves open.

The need for such a module becomes especially clear once one notices that different open questions have very different logical status. The absence of a Standard Model dark-matter candidate is not the same kind of issue as the hierarchy problem. The existence of neutrino masses is not the same kind of issue as the unexplained pattern of quark masses and mixing angles. A central aim of the present note is to train students to distinguish these categories rather than collecting all unresolved questions into one undifferentiated list.

## 1.2 From Module 5 and Module 6 to Module 7

The position of Module 7 inside the course is deliberate. Module 5 established that the Standard Model can reconcile local gauge symmetry with the observed masses of weak gauge bosons and charged fermions through the Higgs mechanism and Yukawa interactions. Module 6 then translated the formal Lagrangian into physical processes: Feynman diagrams, decay widths, cross sections, branching ratios, and measurable observables. Taken together, these modules make the Standard Model visible both as a constrained field theory and as a phenomenological framework connected to experiment.

Module 7 grows naturally from that foundation. Once the student understands where masses, couplings, amplitudes, and observables come from, it becomes meaningful to ask which of those structures are merely inputs, which are explained by symmetry, and which point beyond the theory. The Higgs sector itself already hints at later questions: why are the Yukawa couplings so hierarchical, why are neutrinos so much lighter than the charged fermions, and why is the electroweak scale apparently vulnerable to high-energy quantum corrections? Likewise, the phenomenology of weak decays and flavour processes leads naturally to CP violation, to the CKM matrix, and then to the question of whether the Standard Model can explain the matter-dominated Universe.

In this way Module 7 is not an abrupt digression. It is the conceptual continuation of the previous modules. It takes seriously the lesson that the Standard Model is highly structured, and then asks whether all important features of nature fit comfortably inside that structure.

## 1.3 What this module will and will not do

This module will do four things. First, it will present the Standard Model as a successful but incomplete theory rather than as a failed one. Second, it will explain how CP violation arises within the quark sector through flavour mixing, and why this success is nevertheless insufficient for baryogenesis. Third, it will examine several major open questions — neutrino masses, flavour hierarchies, dark matter, strong CP, and naturalness — in a way that is scientifically serious but pedagogically bounded. Fourth, it will organise these questions into categories so that students see not just *what* the open problems are, but *what kind* of open problems they are.

The module will not attempt a full survey of beyond-the-Standard-Model theories. It will not become a full flavour-physics or neutrino-oscillation course, and it will not attempt a full cosmology chapter. In particular, detailed kaon-mixing formalisms, full PMNS phenomenology, full baryogenesis calculations, and extended model catalogues such as supersymmetry, grand unification, extra dimensions, or composite-Higgs scenarios are outside the scope of the present note. These topics are important, but they belong either to later specialised study or to advanced research-oriented courses.

The guiding discipline of the module is therefore selective depth. Each chosen topic should illuminate a genuine boundary of the Standard Model, but the note must remain recognisably a module within *The Standard Model* course rather than a replacement for a dedicated beyond-the-Standard-Model programme.

## 1.4 Roadmap of the note

The note begins by characterising the Standard Model as a theory of extraordinary success whose limitations are nevertheless already visible. It then turns to discrete symmetries and the role of CP, first at a qualitative level and then in the specific context of flavour physics and the CKM matrix. From there it moves to the phenomenological manifestations of CP violation and to the important conclusion that Standard Model CP violation is too small, in practice, to explain the observed baryon asymmetry.

After this flavour and CP thread, the note widens its scope. It examines the flavour puzzle itself, the evidence for neutrino masses, the bounded logic of Dirac versus Majorana possibilities, and the reason neutrinos already signal incompleteness of the minimal Standard Model. The note then treats the strong CP problem, dark matter, and naturalness. A short section on gravity and other external boundaries follows, not to absorb the module into cosmology or quantum gravity, but to place the Standard Model in a wider conceptual setting.

The final part of the note classifies the different kinds of incompleteness encountered, offers a deliberately bounded outlook toward physics beyond the Standard Model, and then connects Module 7 forward to later phenomenology, data analysis, and research questions. The overall aim is that the student should finish the module with a cleaner conceptual map: not simply a list of mysteries, but an ordered understanding of why the Standard Model is both a triumph and a boundary.

## 2 The Standard Model as a successful but incomplete theory

The Standard Model is best approached as a theory with two simultaneous faces. On the one hand, it is a triumph of relativistic quantum field theory and symmetry-based construction. On the other hand, it is visibly not the end of the story. The first task of this module is therefore to describe success and incompleteness in the same breath, without confusing admiration for the theory with the claim that nothing essential remains unresolved.

### 2.1 Precision success and explanatory power

The Standard Model unifies a remarkable range of phenomena within the gauge structure $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, together with the Higgs mechanism and the observed fermion content. It explains why the weak interaction is chiral, why the photon remains massless after electroweak symmetry breaking, why flavour-changing neutral currents are suppressed, and why many collider observables are calculable and testable. It is hard to overstate the significance of this success: the Standard Model is not a loose collection of rules, but a coherent theory whose pieces constrain one another.

In pedagogical terms, this means that Module 7 must not begin by undermining the previous modules. The right starting point is that the Standard Model deserves the authority it has earned. Only after that authority is recognised does it become meaningful to identify the limits of its explanatory reach.

### 2.2 Why success does not imply completeness

Success does not imply completeness because a theory may be predictive while still containing unexplained structures or while failing to account for some established phenomena. The Standard Model has many free parameters in its gauge, Higgs, and flavour sectors. These parameters can be measured with precision, but the theory does not tell us why they take the values they do. More sharply, some observational facts — most notably nonzero neutrino masses and the existence of dark matter — are not naturally accommodated within the minimal Standard Model picture.

Even when no direct contradiction is present, conceptual tensions remain. The Higgs sector raises naturalness questions about the stability of the electroweak scale. The flavour sector contains striking hierarchies and mixing patterns that appear too structured to be dismissed as random, yet are not predicted from first principles. A mature understanding of the Standard Model therefore includes the recognition that the theory is successful in ways that also expose its boundaries.

### 2.3 Different kinds of open questions

Not all open questions have the same status. Some are *empirical incompletenesses*: a phenomenon exists in nature, but the Standard Model does not provide the corresponding fundamental ingredient. Dark matter is the clearest example. Some are *direct failures of the minimal field content*: neutrino oscillations imply nonzero neutrino masses, yet the minimal renormalisable Standard Model leaves neutrinos massless. Some are *unexplained parameter patterns*: the Yukawa couplings, mass hierarchies, and mixing structures are inserted rather than derived. Others are *conceptual tensions*: the hierarchy problem and strong CP problem are not direct contradictions with experiment, but they place the theory in a sharp explanatory discomfort.

This classification will be revisited later, but it is useful to introduce it already here because it prevents a common pedagogical mistake: treating all unresolved issues as though they were of the same weight and the same logical type.

**Remark 2.1: A limitation is not always a failure of consistency**

A theory may have limitations for very different reasons. Some limitations indicate that known phenomena cannot be accounted for within the theory as it stands; others indicate only that the theory contains unexplained structures or severe tunings. It is therefore important not to use the word *incomplete* too loosely. In this module, the aim is to keep empirical incompleteness, structural incompleteness, unexplained parameters, and conceptual tensions clearly distinct.

### 2.4 Why Module 7 is a bridge rather than a separate BSM course

The present module is designed as a bridge because its aim is diagnostic, not encyclopaedic. It asks why particle physicists are driven beyond the Standard Model, but it stops short of surveying the many possible answers in detail. This is both a scientific and pedagogical choice. Scientifically, the list of possible beyond-the-Standard-Model frameworks is too large and too unsettled for a bounded MSc module of this kind. Pedagogically, students first need a disciplined picture of the questions before they can evaluate proposed answers.

For this reason the present note concentrates on the structure of the open problems themselves. It gives the student a map of the pressure points of the Standard Model, so that later encounters with effective field theory, neutrino model building, dark-matter searches, or hierarchy-motivated extensions can be understood as responses to already well-posed questions.

## 3 Discrete symmetries and where CP enters

Before discussing CP violation as a concrete limitation-related topic, it is useful to recall where discrete symmetries enter the Standard Model at all. The weak interaction is special not merely because it changes flavour, but because it treats left-handed and right-handed states differently. This makes the discrete symmetries $P$, $C$, and $CP$ conceptually central rather than decorative.

### 3.1 $C$, $P$, and $T$ at course level

At course level, one may think of parity $P$ as spatial inversion, charge conjugation $C$ as exchange of particles with antiparticles, and time reversal $T$ as reversal of the temporal direction in the symmetry analysis of the theory. These are discrete transformations rather than continuous symmetry groups. They do not generate Noether currents in the same way as continuous symmetries, but they nevertheless powerfully constrain the form of interactions.

In the Standard Model, electromagnetic and strong interactions are vector-like and respect these discrete symmetries to a very high degree in ordinary treatments. The weak interaction is different because it couples only to left-handed fermions and right-handed antifermions. The possibility of violating $P$ and $C$ is therefore built into the chiral structure of the weak sector.

### 3.2 Why parity and charge conjugation fail in weak interactions

The charged weak current has the schematic form

$$\mathcal{L}_{\mathrm{CC}} \sim \bar{\psi}_L \gamma^\mu \psi_L \, W_\mu + \text{h.c.}, \tag{3.1}$$

which already shows that the interaction selects a chiral component of the fermion fields. Because parity exchanges left- and right-handed descriptions, a purely left-chiral interaction cannot remain invariant under parity. Likewise, charge conjugation maps particle states into antiparticle states in a way that is not respected by the observed weak interaction structure. In that sense, weak interactions violate both $P$ and $C$ very strongly.

This fact matters for Module 7 because it explains why one is naturally led to ask whether the combined operation $CP$ might still be respected. Historically and conceptually, CP was a sharper and more subtle question than $P$ alone.

### 3.3 $CP$ as the sharper question for flavour physics

If both $C$ and $P$ fail separately, one may still hope that the combined transformation $CP$ survives as an approximate or exact symmetry. For a time this seemed plausible. In modern language, however, the Standard Model allows physical CP violation through complex phases in the flavour sector. This is not an arbitrary addition; it is tied to the existence of multiple generations and to the mismatch between weak and mass eigenstates.

**Definition 3.1: CP violation**

CP violation means that the laws governing a process are not invariant under the combined transformation of charge conjugation and parity. In the Standard Model, the physically relevant source of such violation in the quark sector is a complex phase in the CKM matrix that cannot be removed by allowed field redefinitions.

This definition already indicates why CP violation belongs in a module on limitations. It is simultaneously a success of the Standard Model — because the theory does accommodate observed quark-sector CP violation — and a point of incompleteness, because that success still does not solve deeper problems such as the origin of flavour or the baryon asymmetry of the Universe.

### 3.4 Short $CPT$ orientation remark

The role of $CPT$ should be kept clear. In local Lorentz-invariant quantum field theory, $CPT$ is expected to be exact. This means that CP violation is closely tied to time-reversal violation in the usual field-theoretic setting. For the purposes of this module, the practical message is simple: the observed failure of CP symmetry in weak processes is not an isolated curiosity, but part of the deeper discrete-symmetry structure of relativistic quantum field theory.

## 4 Flavour structure and the CKM origin of CP violation

The most important internal source of CP violation in the Standard Model lies in the flavour sector of the quarks. This fact makes the flavour discussion unavoidable in Module 7. One cannot understand why CP violation exists, why three generations matter, or why the flavour sector remains puzzling without first understanding how mass eigenstates and weak eigenstates fail to align.

### 4.1 Mass eigenstates versus weak eigenstates

Before electroweak symmetry breaking, the weak interaction acts on quark doublets organised by gauge representation. After symmetry breaking, the Yukawa matrices generate masses, and these mass terms must be diagonalised. The crucial point is that the matrices used to diagonalise the up-type and down-type quarks are not the same. This mismatch leaves behind a nontrivial unitary matrix in the charged-current weak interaction.

The result is that the quark fields that propagate with definite mass are not identical to the quark combinations that couple diagonally in the weak interaction. Flavour mixing is therefore not an afterthought; it is a direct consequence of the structure of Yukawa couplings and mass diagonalisation.

### 4.2 Yukawa matrices and mixing at orientation level

At a bounded level one may write the quark Yukawa sector schematically as

$$\mathcal{L}_Y \supset -\bar{Q}_L Y_d \Phi d_R - \bar{Q}_L Y_u \widetilde{\Phi} u_R + \text{h.c.}. \tag{4.1}$$

After the Higgs field takes its vacuum expectation value, the Yukawa matrices become mass matrices. Diagonalising them requires independent unitary transformations in flavour space. The weak charged current then takes the form

$$\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}}\, \bar{u}_L \gamma^\mu V_{\text{CKM}} d_L\, W_\mu^+ + \text{h.c.}, \tag{4.2}$$

where $u_L$ and $d_L$ denote vectors of up-type and down-type left-handed mass eigenstates.

The point to keep in view is that the CKM matrix is not a separate ingredient glued onto the Standard Model from outside. It is the residue of the fact that the Yukawa sector cannot, in general, be simultaneously diagonalised in both up and down quark sectors.

## 4.3 The CKM matrix

The CKM matrix is a $3 \times 3$ unitary matrix that mixes the down-type quark flavours in charged-current weak interactions. In pedagogical terms, it does two related jobs. First, it quantifies flavour-changing charged-current transitions. Second, through its irreducible complex phase, it provides the quark-sector source of CP violation in the Standard Model.

At the present level, one does not need the full machinery of precision flavour physics. What matters is the logic: a real CKM matrix would allow flavour mixing without CP violation, while a complex CKM matrix whose phase cannot be removed corresponds to physical CP violation.

## 4.4 Why three generations are required for irreducible CKM CP violation

With only two generations, one can have flavour mixing through the Cabibbo angle, but no irreducible CP-violating phase survives after allowed field redefinitions. A third generation changes the counting. Only then does the flavour structure permit a physical complex phase that cannot be rotated away.

**Example 4.1: Why two generations are not enough**

A $2 \times 2$ unitary mixing matrix can always be reduced, by suitable rephasings of the quark fields, to a single real mixing angle. With three generations, this is no longer possible. The parameter counting leaves one physically meaningful complex phase, and that phase is the origin of quark-sector CP violation in the Standard Model.

This is an important conceptual lesson: the existence of three fermion families is not merely a numerical accident in the Standard Model data table. It is tied to the very possibility of CP violation within the quark sector.

## 4.5 The physical meaning of a complex phase

A complex phase becomes physically meaningful only when it cannot be removed by harmless field redefinitions. In the CKM case, the surviving phase affects interference between amplitudes and thereby enters observable CP-violating quantities. The issue is therefore not that some matrix entries look complex in a chosen basis. The issue is that the theory contains a basis-independent obstruction to making the flavour sector entirely real.

## 4.6 A short invariant-level remark: the Jarlskog viewpoint

One convenient way to express the basis-independent character of CKM CP violation is through the Jarlskog invariant,

$$J = \mathrm{Im}\big(V_{ud} V_{ub}^* V_{tb} V_{td}^*\big). \tag{4.3}$$

A nonzero value of $J$ signals physical CP violation. This quantity is pedagogically useful because it makes clear that CP violation is not an artefact of notation. It is also useful because it vanishes if the situation effectively collapses to two generations or if relevant quark masses become degenerate.

The detailed numerical size of $J$ will not be pursued here. For the purposes of Module 7, the main message is simpler: Standard Model CP violation exists, but it is parametrically small in practice and strongly shaped by the flavour structure itself.

# 5 Physical manifestations of CP violation

Once the flavour-sector origin of CP violation is understood, the next step is to ask how it shows up in nature. For a bounded pedagogical treatment, one should not attempt the full formalism of neutral-meson mixing and decay amplitudes. But one should absolutely show the student where CP violation was first seen and why the subject remains experimentally rich.

## 5.1 Neutral kaons as the historic gateway

The neutral kaon system provided the historic gateway to CP violation. The mesons $K^0$ and $\bar{K}^0$ are distinct strong-interaction flavour states, but weak interactions mix them. Because these states oscillate and decay, they provide a sensitive arena in which tiny CP-violating effects become observable.

At course level, the essential lesson is qualitative: the weak interaction does not preserve CP exactly, and this becomes visible in the behaviour of neutral kaons through mixing and decay patterns that cannot be explained by a CP-symmetric weak theory.

## 5.2 Oscillations, mixing, and CP-violating observables at qualitative level

Meson systems such as the neutral kaons and neutral $B$ mesons illustrate an important general point: CP violation often appears not in isolated decay rates alone, but in the interference of mixing and decay amplitudes. The presence of oscillations between flavour states makes the system especially sensitive to small complex phases.

For the purposes of this note, the student should retain the logic rather than the detailed formalism. Weak interactions generate mixing, mixing allows interference, and interference makes phase information experimentally accessible. This is why flavour factories and precision decay experiments became central laboratories for testing the CP structure of the Standard Model.

## 5.3 $B$-meson systems as later precision tests

Although kaons opened the subject historically, modern precision tests of the CKM picture have relied heavily on $B$-meson physics. The reason is that the third generation enters there more directly, and the associated weak phases provide a stringent test of the CKM framework. In that sense, $B$-physics has not replaced kaon physics but extended and sharpened the Standard Model picture of CP violation.

A bounded mention is enough here. The pedagogical point is that Standard Model CP violation is not a single anomaly attached to one process. It is a coherent flavour-sector phenomenon tested across multiple systems.

## 5.4 What has been established experimentally

What has been established is that CP violation in the quark sector is real and that the CKM framework successfully organises a large body of flavour data. This is a genuine success of the Standard Model. Module 7 must preserve that point clearly. The open questions do not arise because the Standard Model failed to describe observed quark-sector CP violation; they arise because the theory does not explain the deeper pattern of flavour parameters and because the magnitude of its CP violation is insufficient for some cosmological purposes.

### 5.5 What this explains and what it does not explain

The Standard Model explains that CP violation can arise through a complex phase in the quark flavour sector, and it successfully connects that statement to observed meson phenomenology. What it does *not* explain is why the flavour sector takes the particular hierarchical form it does, why there are three generations in the first place, and why the resulting CP violation is so limited in its broader cosmological consequences.

**Take-home message**

The quark sector of the Standard Model does contain real, physical CP violation. This is a success, not a loophole. But it is a success with boundaries: the CKM mechanism depends on the unexplained flavour structure of the theory and does not, by itself, resolve deeper questions such as the matter–antimatter asymmetry of the Universe.

## 6 Why Standard Model CP violation is not enough

At this point the module turns from internal success to explanatory shortfall. The key question is not whether CP violation exists in the Standard Model, but whether the amount and structure of that violation can explain why the observable Universe contains far more matter than antimatter.

### 6.1 The matter–antimatter asymmetry question

A Universe with exactly symmetric matter and antimatter production in the early thermal history would not lead naturally to the matter-dominated cosmos we observe. The very existence of stars, planets, and ordinary baryonic matter therefore raises a sharp question: why did a small excess of matter survive over antimatter?

This is not a cosmetic issue. It is one of the places where cosmological observation and particle-physics microdynamics meet. Any credible explanation requires ingredients that are, at least in principle, expressible in terms of particle interactions and their violation of certain symmetries.

### 6.2 Sakharov conditions at course level

At course level, the standard orientation is given by the Sakharov conditions:

- baryon-number violation,
- $C$ and $CP$ violation,
- departure from thermal equilibrium.

These conditions are not a detailed mechanism by themselves. Rather, they tell us what kinds of ingredients are needed if a particle theory is to generate a baryon asymmetry dynamically.

### 6.3 Why the Standard Model falls short

The Standard Model contains some of the needed ingredients in limited form. It has CP violation in the quark sector, and baryon number is not an absolute symmetry once nonperturbative electroweak effects are taken seriously. Yet detailed analyses show that the available CP violation is too small, and the electroweak transition in the minimal Standard Model is not of the kind needed for a sufficiently strong baryogenesis mechanism.

The conclusion is therefore subtle but important. The Standard Model is not absolutely incapable of addressing the question at a formal level; rather, its actual quantitative content appears insufficient. This is exactly the kind of explanatory shortfall that Module 7 is designed to clarify.

### 6.4 Pedagogical boundary: not a full baryogenesis review

The present module will not attempt a full review of baryogenesis or leptogenesis scenarios. Those topics quickly require thermal field theory, nonequilibrium dynamics, and model-dependent new-physics ingredients. Here it is enough to understand the logic of the shortfall: the observed quark-sector CP violation of the Standard Model does not seem capable of explaining the matter-dominated Universe by itself.

## 7 The flavour problem beyond observed CKM success

The flavour sector is one of the least understood parts of the Standard Model. It is rich in measured structure, but poor in explanatory principle. This is why flavour must appear twice in the present module: first as the origin of quark-sector CP violation, and second as a major puzzle in its own right.

### 7.1 Why three generations?

The Standard Model contains three generations of quarks and leptons with identical gauge quantum numbers but very different masses. Nothing within the Standard Model explains why there are three and not one, two, or more. The existence of three generations is empirically given; the theory organises it, but does not derive it.

The fact that three generations are the first number allowing irreducible CKM CP violation is intriguing, but it is not yet an explanation. At best it is a hint that the flavour structure carries meaning deeper than the Standard Model itself reveals.

### 7.2 Yukawa hierarchies and unexplained fermion masses

The Higgs mechanism explains how charged fermion masses are generated once Yukawa couplings are present, but it does not explain the values of those Yukawa couplings. This distinction should remain explicit. In the Standard Model,

$$m_f = \frac{y_f v}{\sqrt{2}}, \tag{7.1}$$

but the theory does not predict the observed pattern of $y_f$.

The fermion masses span a huge range, and the charged-fermion and quark hierarchies look structured rather than random. Yet there is no symmetry principle inside the minimal Standard Model that explains this pattern. The flavour puzzle is therefore not just about mixing matrices; it is about the very origin of mass hierarchies among matter fields.

### 7.3 Why CKM and PMNS look so different

Another striking fact is that quark mixing and lepton mixing look qualitatively different. The CKM matrix is close to diagonal, while the PMNS matrix exhibits comparatively large mixing angles. This contrast is one of the most suggestive features of the flavour sector. It does not by itself prove any specific new-physics framework, but it strongly suggests that the Standard Model flavour story is incomplete.

In the present module, this comparison is used only as a diagnostic observation. A full theory of lepton flavour and neutrino mixing lies outside the current scope.

### 7.4 Free parameters and the flavour puzzle

A substantial fraction of the Standard Model's free parameters belong to the flavour and Higgs sectors. This alone is pedagogically significant. The theory is elegant in its gauge structure, but the flavour sector is largely put in by hand. The open question is not whether the Standard Model can *fit* flavour data; it can. The open question is whether those data reflect a deeper organising principle absent from the theory as presently formulated.

## 8 Neutrino masses as evidence beyond the minimal Standard Model

Neutrino masses occupy a special position among Standard Model limitations because they represent more than a conceptual discomfort. Oscillation experiments show that neutrinos are not all massless, and this fact already forces one beyond the minimal renormalisable Standard Model field-content picture.

### 8.1 Neutrino oscillations imply nonzero neutrino masses

If neutrino flavour states oscillate into one another during propagation, then the flavour states cannot coincide with exact mass eigenstates. Oscillation probabilities depend on differences of neutrino mass-squared eigenvalues and on nontrivial mixing. The conceptual implication is immediate: at least two neutrino masses must be nonzero.

This statement is one of the cleanest examples in the module of an open question grounded directly in experiment. One need not solve the full oscillation formalism here to understand its force. Neutrino oscillations are evidence that the minimal Standard Model picture of massless neutrinos is incomplete.

### 8.2 Why the minimal Standard Model leaves neutrinos massless

In the minimal renormalisable Standard Model, no right-handed neutrino field is included, and therefore no renormalisable Yukawa term analogous to the charged-fermion case can be written for neutrinos. As a result, neutrinos remain massless in that minimal field-content description.

This is an important pedagogical contrast with the charged-fermion sector. For electrons and quarks, the Standard Model supplies a renormalisable mass-generation mechanism through Higgs couplings. For neutrinos, the minimal theory does not. The incompleteness is therefore structural rather than merely numerical.

### 8.3 Dirac versus Majorana possibilities at bounded level

Once one accepts that neutrinos are massive, one must ask what kind of mass they have. A Dirac mass would require right-handed neutrino fields and extraordinarily small Yukawa couplings. A Majorana mass would identify neutrinos, in an appropriate sense, with their own antiparticles and would violate lepton number.

The present module will not decide between these possibilities. It is enough here to note that both options lie beyond the minimal renormalisable Standard Model picture and that the distinction matters conceptually for the structure of particle theory.

### 8.4 The dimension-five / seesaw orientation remark

A particularly elegant orientation-level statement is that neutrino masses can arise from the effective dimension-five operator

$$\mathcal{L}_{\nu,\text{eff}} \sim \frac{1}{\Lambda}(L\Phi)(L\Phi) + \text{h.c.}, \tag{8.1}$$

which, after electroweak symmetry breaking, gives a small Majorana mass of order $v^2/\Lambda$. This already suggests why neutrino masses may naturally be much smaller than charged-fermion masses. The seesaw idea is the most famous realisation of this logic, but a full treatment is beyond the present note.

### 8.5 What is established and what remains open

What is established is that neutrinos oscillate and therefore that neutrino masses and lepton mixing are real. What remains open includes the absolute mass scale, the mass ordering, the Dirac-versus-Majorana question, the size and origin of leptonic CP violation, and the deeper mechanism behind neutrino mass generation. This combination of firm evidence and unresolved structure makes neutrinos a paradigmatic Module 7 topic.

## 9 The strong CP problem

The Standard Model also contains a more internal and conceptually sharp puzzle: why does QCD appear to respect CP so extremely well when its Lagrangian seems to allow a CP-violating term? This is the strong CP problem. Unlike dark matter or neutrino masses, it is not primarily driven by missing matter content or by a direct contradiction with the minimal field list. Instead it is a parameter-naturalness puzzle of unusual sharpness.

### 9.1 The theta term in QCD

The QCD Lagrangian may contain the term

$$\mathcal{L}_\theta = \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}. \tag{9.1}$$

Once the quark-mass phases are treated properly, the physically relevant parameter is an effective angle usually denoted by $\bar{\theta}$. If this parameter were of order unity, strong-interaction CP violation would be observable.

### 9.2 Why the problem is physical

The strong CP problem is physical because the relevant angle cannot simply be dismissed as a removable convention once the full flavour structure is taken into account. In other words, the issue is not just that a symbol $\theta$ can be written in the Lagrangian. The issue is that a measurable CP-violating effect in QCD is in principle allowed and yet appears to be astonishingly suppressed.

### 9.3 Neutron electric dipole moment and the smallness puzzle

The experimental proxy for this suppression is the neutron electric dipole moment. A nonzero strong CP angle would induce an electric dipole moment far larger than the present experimental bound unless $\bar{\theta}$ were extremely tiny. The resulting inference — that $\bar{\theta}$ must be extraordinarily small — is what makes the problem so sharp. Nothing inside the Standard Model explains naturally why this parameter should be so close to zero.

**Remark 9.1: Why strong CP is a different kind of limitation**

The strong CP problem is not the same kind of issue as dark matter or neutrino masses. It does not begin from a missing particle species or from an experimentally established mass term absent in the minimal theory. Instead, it begins from the opposite tension: the theory seems to allow a CP-violating parameter that experiment forces to be unnaturally tiny. It is therefore best classified as a sharp conceptual and parameter-naturalness problem.

### 9.4 A very short remark on axions only as a bridge

One famous response to the strong CP problem introduces a new dynamical field whose low-energy consequence is an axion-like degree of freedom. This idea is historically and scientifically important, but it belongs to the beyond-the-Standard-Model response rather than to the diagnostic task of the present module. It is therefore mentioned only as a bridge, not developed as a full topic.

## 10 Dark matter and missing matter content

Dark matter provides one of the clearest empirical reasons to believe that the Standard Model is incomplete. It is not merely a theoretical preference or an aesthetic complaint. It is driven by multiple lines of astrophysical and cosmological evidence that point to a substantial gravitating matter component not accounted for by the known Standard Model particles.

### 10.1 Why dark matter is not optional phenomenology

The existence of dark matter is inferred from galactic rotation curves, large-scale structure, gravitational lensing, and precision cosmological data. The cumulative picture is that luminous baryonic matter cannot account for the observed gravitational behaviour of the Universe across many scales. For the purposes of this module, the detailed observational analysis is not required. What matters is the conceptual conclusion: there exists strong evidence for matter whose dominant observed role is gravitational, but whose identity is not supplied by the Standard Model.

### 10.2 Why the Standard Model does not provide an adequate dark-matter candidate

The Standard Model contains neutral particles, most obviously the neutrinos, but none of the known particles provide the right combination of abundance, clustering behaviour, and stability properties required for the dominant dark matter inferred in cosmology. The Standard Model therefore lacks an adequate dark-matter candidate in its known particle content.

### 10.3 Why neutrinos are not enough

Neutrinos are real, neutral, and weakly interacting, so they are a natural first thought. However, they are too light and therefore behave as hot or relativistic dark matter in cosmological structure formation, whereas the dominant dark matter required by observations must be cold or at least much less relativistic during the relevant eras of structure growth. Thus neutrinos may contribute a small matter component, but they do not solve the dark-matter problem.

### 10.4 Experimental and observational motivation at a bounded level

The present note will not become a cosmology chapter or a dark-matter search review. It is enough to state the Standard Model lesson clearly: dark matter is not an optional speculative embellishment but a major empirical sign that known Standard Model matter content is incomplete. Any future theory claiming to improve on the Standard Model must confront this fact.

## 11 Naturalness, hierarchy, and the electroweak scale

The next limitation-type topic is different again. The hierarchy problem is not a direct observational contradiction in the same way as neutrino masses or dark matter. Rather, it is a conceptual tension concerning the stability of the Higgs-sector scale under quantum corrections.

### 11.1 Why the Higgs sector makes this issue visible

The Higgs field is special because it is the only known fundamental scalar field in the Standard Model. Scalar masses are notoriously sensitive to high-energy physics in a way that fermion masses and gauge symmetries often are not. This means that the electroweak scale, encoded in the Higgs potential and vacuum expectation value, seems unusually vulnerable to ultraviolet physics.

The relevance of Module 5 should therefore be recalled here. Once the student understands that the Higgs sector is structurally necessary for electroweak mass generation, it becomes natural to ask whether the parameters of that same sector are themselves stable and natural.

### 11.2 Quantum corrections and scale sensitivity at course level

At a schematic level, quantum corrections to the Higgs mass parameter may be written as

$$\delta m_H^2 \sim \frac{\Lambda^2}{16\pi^2} \times (\text{couplings and signs}), \tag{11.1}$$

where $\Lambda$ denotes a high-energy scale at which new physics or an ultraviolet description becomes relevant. The precise coefficient and regularisation details are not the focus here. What matters is the structural message: the Higgs mass parameter is highly sensitive to high scales unless some mechanism controls or cancels this sensitivity.

### 11.3 What physicists mean by "naturalness"

In this context, *naturalness* means, roughly, that the observed low-energy parameters of a theory should not require extremely delicate cancellations among unrelated high-energy contributions unless there is a good reason. A theory is felt to be unnatural when a small observed number results only from a large cancellation between much larger ingredients with no protecting principle.

Naturalness is therefore a judgement about explanatory quality rather than a theorem. It is powerful because it organises much of modern beyond-the-Standard-Model thinking, but it should not be presented as identical to an experimental exclusion.

### 11.4 Why this is a conceptual tension rather than a simple observational contradiction

The Standard Model with a light Higgs boson is not experimentally contradicted by the hierarchy issue. The observed Higgs boson exists, and the theory can be used successfully at accessible energies. The discomfort is conceptual: why should the electroweak scale remain so much smaller than much higher candidate scales without an additional organising principle? This is why the hierarchy problem should be presented as a conceptual tension rather than as a direct empirical failure.

## 12 Other boundaries of the Standard Model

Beyond the central Module 7 topics, it is useful to mention a few additional boundaries of the Standard Model in a carefully controlled way. These remarks are not meant to expand the module into quantum gravity or cosmology, but to prevent the false impression that the Standard Model is intended as a complete theory of all fundamental phenomena.

### 12.1 Gravity as external to the Standard Model

Gravity is not included in the Standard Model gauge structure. At ordinary laboratory and collider energies this omission is practically harmless because gravitational interactions are negligible compared with the gauge interactions. Nevertheless, as a matter of fundamental scope, the Standard Model is not a complete theory of all interactions. A full ultraviolet quantum theory incorporating gravity remains unknown.

### 12.2 Cosmological constant / dark energy as a bounded remark

The accelerated expansion of the Universe points to dark energy or a cosmological constant whose small observed value is not explained by Standard Model particle physics. This is an extremely deep problem, but it lies well beyond the proper scope of the present module. It is mentioned here only to situate the Standard Model within a larger landscape of incompleteness.

### 12.3 Strong-coupling limits of calculability versus true theory failure

One final distinction is worth stating explicitly. Sometimes the difficulty lies not in the theory itself, but in our ability to extract quantitative results from it. Low-energy QCD is the clearest example: confinement and strong coupling make calculations difficult, but this does not imply that QCD is conceptually false. Difficulty of calculation should therefore not be confused with failure of the Standard Model. This distinction matters when classifying limitations responsibly.

## 13 What kinds of incompleteness have we encountered?

By this stage the note has accumulated several different open problems. It is now pedagogically valuable to classify them explicitly. Doing so helps students avoid conflating all unresolved questions into a single vague dissatisfaction with the Standard Model.

### 13.1 Empirical incompleteness

Empirical incompleteness refers to cases where observations demand physical content not adequately supplied by the Standard Model. Dark matter is the clearest example. The phenomenon is supported by substantial observational evidence, yet the Standard Model does not provide the required dominant matter component.

### 13.2 Structural incompleteness

Structural incompleteness refers to cases where the minimal Standard Model field content and renormalisable interaction structure cannot reproduce an established fact. Neutrino masses belong here. The evidence for oscillations is clear, and the minimal Standard Model leaves neutrinos massless. Something must be added or modified.

### 13.3 Unexplained parameter patterns

The flavour puzzle — masses, mixing angles, and Yukawa hierarchies — is best classified as an unexplained parameter-pattern problem. The Standard Model accommodates these data, but does not derive them. The pattern is structured enough to look meaningful, yet the theory provides no deeper reason for it.

### 13.4 Conceptual tensions

Naturalness and the strong CP problem are best classified as conceptual tensions or sharp explanatory puzzles. They do not invalidate the Standard Model at accessible energies, but they make some of its parameter choices look deeply unnatural or insufficiently motivated.

### 13.5 Why this classification matters pedagogically

This classification matters because different kinds of incompleteness motivate different kinds of research strategy. Empirical incompleteness encourages searches for missing particle content or interactions. Structural incompleteness motivates extensions of the field content or effective operators. Parameter-pattern puzzles motivate symmetry-based flavour theories. Conceptual tensions motivate protective principles, ultraviolet structures, or dynamical selection mechanisms. A student who sees these distinctions clearly is in a much better position to understand why particle physicists pursue such different beyond-the-Standard-Model directions.

## 14 A bounded outlook toward physics beyond the Standard Model

Having identified the main pressure points of the Standard Model, it is natural to ask how one should think about what comes next. The present section is intentionally bounded. It does not survey detailed models, but gives only the minimum orientation needed to understand how Module 7 leads toward modern research questions.

### 14.1 Effective-field-theory viewpoint

One useful perspective is that the Standard Model may itself be viewed as an effective field theory valid up to some scale, with higher-dimensional operators encoding the low-energy imprint of more fundamental physics. In this language, neutrino masses, flavour effects, electric dipole moments, and rare processes can all be understood as possible windows into degrees of freedom not yet directly seen.

The effective-field-theory viewpoint is pedagogically valuable because it allows one to think systematically about incompleteness without committing prematurely to one specific ultraviolet completion.

### 14.2 What kinds of new-physics questions are motivated

The questions motivated by Module 7 include: what generates neutrino masses, what constitutes dark matter, what explains flavour hierarchies and mixing patterns, what controls the Higgs-sector scale, and what mechanism suppresses strong CP violation. Each of these questions can be approached through many candidate theories, but the important point for the present note is that they arise from different kinds of tension with the Standard Model and should not be collapsed into a single model-building slogan.

### 14.3 Why the present module stops before a full model survey

A full model survey would require introducing a large catalogue of frameworks — supersymmetry, grand unification, extra dimensions, axion models, composite Higgs ideas, and many others — together with their phenomenology and current constraints. That would immediately change the character of the module. The present note deliberately stops earlier. It aims to deliver a disciplined understanding of the problems, leaving detailed study of proposed solutions to later specialised contexts.

## 15 What Module 7 will not attempt

The preceding sections already contain the core conceptual content of Module 7. The present section simply marks nearby topics that are important, but should remain outside the present note's main development.

### 15.1 Full BSM model catalogues

Modern particle theory contains a vast range of proposals extending the Standard Model. These models are important in research, but listing them without first stabilising the underlying questions would dilute the pedagogical purpose of the present module. Here it is enough to understand the pressure points that make such extensions plausible or attractive.

### 15.2 Full neutrino-oscillation phenomenology

The phenomenology of neutrino oscillations includes matter effects, long-baseline experiments, global fits of mixing angles and mass splittings, and prospective leptonic CP-violation measurements. These are fascinating topics, but they belong to a dedicated neutrino module or course. In the present note, neutrinos are discussed only to the extent needed to show that the minimal Standard Model is incomplete.

### 15.3 Detailed flavour-factory formalism

The precision machinery of kaon, charm, and $B$-physics — including mixing matrices, time-dependent CP asymmetries, hadronic uncertainties, and global CKM fits — lies beyond the present scope. Module 7 needs enough flavour physics to explain the origin and significance of CP violation, but not enough to become a full flavour-dynamics course.

### 15.4 Full cosmology

Baryogenesis, dark matter, dark energy, inflation, and the thermal history of the early Universe are deeply connected to Standard Model limitations. Yet a complete treatment would demand a separate course in cosmology or astroparticle physics. Here these issues are used only insofar as they expose the boundaries of the Standard Model as a particle-physics theory.

# 16 Bridge to later modules

With the boundaries of the Standard Model now visible, the natural next step is to understand how these conceptual questions connect to phenomenology, searches, and research practice.

## 16.1 From open questions to data analysis and searches

Many of the open questions discussed here are not purely abstract. Dark matter motivates direct, indirect, and collider searches. Neutrino masses motivate oscillation experiments and rare-process studies. CP violation motivates precision flavour measurements and electric-dipole-moment searches. Naturalness questions motivate searches for new particles or new dynamics associated with the electroweak scale. Thus Module 7 points not only toward theory, but also toward the observational strategies by which particle physics tests its own future.

## 16.2 From formal incompleteness to research programmes

A major educational purpose of Module 7 is to convert vague awareness of “things the Standard Model does not explain” into a sharper sense of real research programmes. Once students can distinguish empirical incompleteness from conceptual tension, they are better prepared to understand why different experiments and theories pursue very different targets.

## 16.3 Position of Module 7 inside the full AGH course logic

Within the full AGH course logic, Module 7 has a capstone role. Earlier modules developed the language, symmetry logic, relativistic fields, gauge structure, QCD, electroweak symmetry breaking, and phenomenology. Module 7 closes that arc by asking what remains unresolved after one has understood the Standard Model on its own terms. It therefore acts as the conceptual bridge between Standard Model mastery and the wider research landscape of modern particle physics.

# 17 Final summary and conceptual map

## 17.1 The logical chain of Module 7

The logic of the module can be summarised in one chain. The Standard Model is a highly successful gauge quantum field theory, but its discrete-symmetry structure and flavour sector reveal that CP violation is possible. Quark-sector CP violation arises through CKM mixing and requires three generations. This explains observed CP-violating phenomena in flavour physics, yet does not suffice to account for the baryon asymmetry of the Universe. Meanwhile, neutrino oscillations imply neutrino masses beyond the minimal Standard Model picture, the flavour sector contains unexplained hierarchies and mixing patterns, QCD appears to contain a finely suppressed CP-violating parameter, dark matter is absent from the Standard Model particle content, and the Higgs sector raises naturalness questions. These facts do not erase the Standard Model’s success; they define its frontier.

### 17.2 The Standard Model as both triumph and boundary

A good theory is not discredited by having boundaries. On the contrary, one mark of a deep theory is that its successes are sharp enough to make its unresolved questions equally sharp. The Standard Model is precisely such a theory. It describes an enormous body of laboratory and collider physics with extraordinary success, yet it also leaves behind structured mysteries that are too important to ignore. Module 7 is therefore not an anti-Standard-Model chapter. It is the chapter in which the student learns to take the Standard Model seriously enough to see where it stops.

### 17.3 What the student should now be able to see

After this module, the student should be able to explain why CP violation belongs to the flavour sector of the Standard Model, why three generations matter for quark-sector CP violation, why observed CP violation does not solve the baryon asymmetry problem, why neutrino masses already signal incompleteness of the minimal Standard Model, why dark matter and strong CP pose different kinds of challenge, and why naturalness is a conceptual tension rather than a direct observational contradiction. The student should also be able to classify open questions by type and to understand why this classification matters for later theoretical and experimental work.

**Take-home message**

The Standard Model should be understood as both a triumph and a boundary. It successfully explains an enormous range of particle phenomena, including observed quark-sector CP violation, yet it leaves crucial questions unresolved: the origin of flavour hierarchies, the smallness of strong CP violation, the existence of neutrino masses, the identity of dark matter, the matter–antimatter asymmetry of the Universe, and the apparent instability of the electroweak scale. Module 7 is therefore not a catalogue of speculative models, but the disciplined point at which Standard Model mastery becomes awareness of its limits.

## A Conventions and notation summary

| Quantity | Convention |
|---|---|
| Metric | $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ |
| Natural units | $\hbar = c = 1$ |
| Quark mixing matrix | $V_{\mathrm{CKM}}$ |
| Lepton mixing matrix | $U_{\mathrm{PMNS}}$ |
| Baryon and lepton number | $B, L$ |
| Higgs vacuum expectation value | $v$ |
| Strong CP angle | $\bar{\theta}$ |
| Dark-matter abundance symbol | $\Omega_{\mathrm{DM}}$ |

*Convention note*: in this note, neutrino mixing notation is used only for orientation. The present module is not a full PMNS or oscillation-phenomenology treatment.

## B Discrete-symmetry and CP-violation summary

The conceptual chain for discrete symmetries in the Standard Model is:

- the weak interaction is chiral and therefore violates $P$ and $C$ strongly;
- the sharper question is whether $CP$ is conserved;
- in the quark sector, physical CP violation arises from an irreducible complex phase in $V_{\text{CKM}}$;
- at least three generations are required for such a phase to remain after field redefinitions;
- a basis-independent measure of quark-sector CP violation is the Jarlskog invariant,

$$J = \operatorname{Im}\bigl(V_{ud} V_{ub}^* V_{tb} V_{td}^*\bigr). \tag{B.1}$$

## C CKM / flavour-structure summary

The flavour logic of the quark sector can be summarised as follows:

$$\mathcal{L}_Y \supset -\bar{Q}_L Y_d \Phi d_R - \bar{Q}_L Y_u \widetilde{\Phi} u_R + \text{h.c.}, \tag{C.1}$$

$$\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}}\, \bar{u}_L \gamma^\mu V_{\text{CKM}} d_L\, W_\mu^+ + \text{h.c.}. \tag{C.2}$$

After electroweak symmetry breaking, the up- and down-type mass matrices are diagonalised separately. Their mismatch leaves the CKM matrix in the charged-current weak interaction. This explains flavour mixing but not the deeper origin of the Yukawa hierarchies or why the CKM matrix is nearly diagonal.

## D Neutrino-mass and oscillation-summary note

The minimal Standard Model leaves neutrinos massless. Yet oscillation experiments imply nonzero neutrino masses and nontrivial lepton mixing. A useful orientation formula is the effective dimension-five operator

$$\mathcal{L}_{\nu,\text{eff}} \sim \frac{1}{\Lambda}(L\Phi)(L\Phi) + \text{h.c.}, \tag{D.1}$$

which yields neutrino masses of order $v^2/\Lambda$ after electroweak symmetry breaking. This appendix is only a reminder of the logic; a full treatment of oscillation phenomenology, matter effects, mass ordering, and leptonic CP violation lies beyond the present module.

# E Open-question classification summary

| Category | Representative Module 7 example |
|---|---|
| Empirical incompleteness | Dark matter: strong evidence for gravitating matter not supplied by the Standard Model particle content |
| Structural incompleteness | Neutrino masses: oscillations imply nonzero masses absent in the minimal renormalisable Standard Model picture |
| Unexplained parameter patterns | Yukawa hierarchies, fermion masses, CKM/PMNS patterns, and the question of why there are three generations |
| Conceptual tension | Naturalness of the Higgs-sector scale and the strong CP problem |
| Scope limitation | Gravity and dark energy lie outside the Standard Model as a particle-physics theory |

# F Guided-check summary

**Guided checks**

- Can you explain why three generations are required for irreducible CKM CP violation?
- Can you state clearly the difference between observed quark-sector CP violation and the baryon asymmetry problem?
- Can you explain why neutrino oscillations imply that the minimal Standard Model is incomplete?
- Can you distinguish dark matter as empirical incompleteness from naturalness as conceptual tension?
- Can you explain why the strong CP problem is not simply another version of the CKM CP-violation story?
- Can you classify the main Module 7 topics into empirical incompleteness, structural incompleteness, unexplained parameter patterns, and conceptual tensions?